A CONCEPTUAL AND ANALYTICAL STUDY

OF THE UTILITY OF SPEED IN NAVAL OPERATIONS.

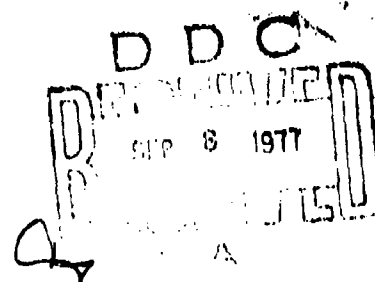### VOLUME II - APPENDIX

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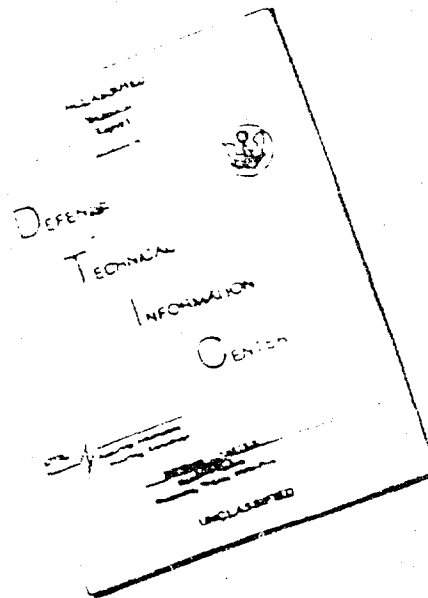Department of the Navy
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APPENDIX

APPENDIX - DETAILED GEOMETRY, DERIVATIONS AND CALCULATIONS

A.   TRANSIT

1.   Base Loss Factor[*]

This section derives the base loss factor (BLF) equation. The base loss factor determines the number of platforms required to keep one platform on station.

The general BLF is the cycle time of a platform divided by the time on station. A cycle is defined as the time interval between overhauls.

The following definitions are necessary to develop the general BLF equation. All times are in months.

Definitions

T_V = overhaul time

T_{Tr} = two-way transit time

T_{St} = time on station

T_{Cy} = total times in a cycle

$b = \dfrac{T_{Cy}}{T_{Cy} - T_V}$ = overhaul coefficient ($^1/b$ is fraction of platforms out-of-overhaul)

n = on station requirements in platforms

N = total requirements to support n platforms on station

$N' = \dfrac{N}{b}$ = number of platforms out-of-overhaul and available for operation

nT_{Cy} = on station requirements in platform months/cycle

$N'T_{Cy}$ = platform months available from out-of-overhaul platforms to provide on station requirements

N_M = fraction of total platforms out of overhaul which are in upkeep

$N_M N'T_{Cy}$ = total upkeep time in platform months/cycle

N_T = training coefficient (fraction of non-deployed, out-of-overhaul time given to self training)

$N_T (N'T_{Cy} - nT_{Cy})$ = normal self training time for non-deployed, out-of-overhaul platforms

$\dfrac{nT_{Cy}}{T_{St}}$ = number of roundtrip transits/cycle from base to station

$\dfrac{nT_{Cy} T_{Tr}}{T_S}$ = number of platform months/cycle spent in transit

The out of overhaul time per cycle is equal to the sum of the on station, upkeep, training and transit time.

$$N'T_{Cy} = nT_{Cy} + N_M N'T_{Cy} + N_T (N'T_{Cy} - nT_{Cy}) + \frac{nT_{Cy} T_{Tr}}{T_{St}} \qquad \text{A-1}$$

Dividing out T_{Cy} and collecting common terms:

$$N' - N_M N' - N_T N' = n - N_T n + \frac{nT_{Tr}}{T_{St}} \qquad \text{A-2}$$

$$\frac{N'}{n} = \frac{1 + \dfrac{T_{Tr}}{T_{St}} - N_T}{1 - N_M - N_T}$$

and,

$$BLF = \frac{N}{n} = b \left[\frac{1 + \dfrac{T_{Tr}}{T_{St}} - N_T}{1 - N_M - N_T} \right] \text{, the generalized BLF equation.} \qquad \text{A-3}$$

In Section III of the report, only ready platforms are considered. Said another way, the platforms are assumed to be out-of-overhaul, already maintained and trained so that the out-of-overhaul, upkeep and training times are effectively zero and the equation reduces to (see following page):

$$BLF = 1 + \frac{T_{Tr}}{T_{St}}$$

$$= \frac{T_{St} + T_{Tr}}{T_{St}} \qquad\qquad \text{A-4}$$

This equation represents the number of ready platforms needed to keep one on station.

The impact of transit speeds and platform endurance on force level requirements can be evaluated by this BLF equation. In this approach, the total endurance time, T_E, is defined as the two-way transit time plus the on station time and is given by the expression

$$T_E = T_{Tr} + T_{St}$$

Then the BLF can be written as

$$BLF = \frac{\dfrac{V_{Tr} T_E}{D}}{\dfrac{V_{Tr} T_E}{D} - 1} = \frac{\lambda}{\lambda - 1} \qquad\qquad \text{A-5}$$

where   D = two way transit distance

     V_{Tr} = transit speed

     $\lambda = \dfrac{V_{Tr} T_E}{D}$

The total endurance time can be written as

$$T_E = \frac{D}{V_{Tr}} \left(\frac{BLF}{BLF - 1} \right) \qquad\qquad \text{A-6}$$

For any given transit distance, the BLF is reduced by increasing the product of the total endurance time and the transit speed. The overall change in force level requirements which results from increasing transit speed depends on the effect on platform endurance from that increased speed. When endurance is relatively independent of speed, increased transit speed will result in reduced force level requirements.

In the case where the platform endurance is a dependent function of its speed, the overall effect of increased transit speeds on force level requirements is not immediately apparent. This effect can also be evaluated by the BLF equations. In this discussion the endurance of platforms using conventional propulsion is assumed to be limited by the amount of fuel they can carry (assuming that refueling is possible only at the origin).

When fuel is the quantity that limits endurance, the time on station can be defined as

$$T_s = \frac{\text{Fuel Available On Station}}{\text{Fuel Consumption Rate On Station}}$$

The fuel available on station is the difference between the fuel that was available at the origin and the fuel necessary for two-way transit.

Fuel available on station $= F - \frac{D}{V_{Tr}} r_{Tr}$

where

F = fuel available at origin

r_{Tr} = fuel consumption rate while in transit

Thus,

$$T_s = \frac{F - \frac{D}{V_{Tr}} r_{Tr}}{r_{st}}$$

A-7

where

r_{st} = fuel consumption rate while on station

The total endurance time T_E, is approximated by the expressions:

$$T_E = T_{Tr} + T_{St}$$

$$= \frac{D}{V_{Tr}} + \frac{F - \frac{D}{V_{Tr}} r_{Tr}}{r_{St}} \qquad \text{A-8}$$

The base loss factor can then be written as

$$BLF = \frac{T_{Tr} + T_{St}}{T_{St}}$$

$$= 1 + \frac{\frac{D}{V_{Tr}} r_{St}}{F - \frac{D}{V_{Tr}} r_{Tr}} \qquad \text{A-9}$$

when fuel limits the endurance of the platform.

2. Transit to Destination

This section derives the equations used for the economic analysis
of a single platform transiting from an origin to a destination. The total
transportation cost includes all costs associated with transporting the
cargo from an origin to a destination and is the sum of three component
costs:

— the dollar value of the cargo which could alternately be invested
   at some rate of interest during the time of transit

— the cost associated with operating the platform (speed
   independent)

— a speed dependent cost related to energy consumption

The value of the cargo at the origin is the number of tons of cargo times
the value per ton of cargo. The value per ton of cargo can be expressed as the
dollar value of the cargo or weighted dollar value when the cargo has a worth
beyond the market value. The cargo value could be alternately invested during
the time of transportation from the origin to the destination. The portion of
the total transportation cost which is assigned to the cargo itself is the cargo
value times the investment rate times the transit time.

The transportation costs due to the particular platform used are divided
into the platform operating costs (speed independent) and the energy consump-
tion costs (speed dependent). The platform operating costs include deprecia-
tion of the platform and equipment, personnel costs, maintenance, port fees,
overhaul and special costs due to the particular exercise. These operating
costs can be added together and divided by the product of the lifetime operating
hours of the platform and its cargo capacity to obtain an average platform

operating cost per ton hour.* These costs were assumed to be independent of speed for this study. Some of these costs would become speed dependent if the platform utilization varied because of changes in speed.

The speed dependent costs were identified as being chiefly related to energy consumption. Energy consumption is a function of the propulsion system and the mode of transport.

The transportation cost is given by the expression:

$$\text{Transportation Cost} = CQIT + C_0QT + kV^\alpha DQ$$

where:

C = cost of cargo (dollars/ton)

Q = number of tons of cargo

I = investment rate (%/hour)

T = time to transit from origin to destination (hours)

C_0 = operating cost (dollars/ ton·hour)

V = speed of transit (knots)

k = proportionality constant relating speed to fuel consumption

α = proportionality constant relating fuel consumption to mode of transit

kV^α = energy consumption cost/ton·mile

D = distance from origin to destination

and

$$\frac{\text{Transportation Cost}}{\text{Ton·Mile}} = (CI + C_0)\frac{1}{V} + kV^\alpha \qquad \text{A-10}$$

The sender would like to minimize the total transportation cost per ton mile by choosing a V such that the transportation cost per ton-mile is a minimum.

*References for operating cost data include The Utility of High Performance Watercraft for Selected Missions of the United States Coast Guard (U), Project 721530, Center for Naval Analysis, November 1972.

Differentiating:

$$\frac{d\left(\frac{\text{Transportation Cost}}{\text{Ton} \cdot \text{Mile}}\right)}{dV} = \frac{-(CI + C_o)}{V^2} + \alpha k V^{\alpha-1} = 0$$

$$V_{opt}^{\alpha+1} = \frac{CI + C_o}{k\alpha}$$

$$V_{opt} = \left(\frac{CI + C_o}{k\alpha}\right)^{\frac{1}{\alpha+1}} \qquad\qquad \text{A-11}$$

where V_{opt} is the speed that minimizes the cost of transporting the cargo.
When CI is much smaller than C_o, then

$$V_{opt} \approx \left(\frac{C_o}{k\alpha}\right)^{\frac{1}{\alpha+1}} \qquad\qquad \text{A-12}$$

3. Sustained Logistic Support

This section develops the equations for the number of platforms required
to fill a pipeline.

Suppose that Q tons of cargo must be delivered during a time period, T.
The average rate of delivery must be

$$\frac{Q}{T} = \frac{\text{Tons}}{\text{Unit Time}}$$

Let each platform have a payload capacity of Q_p tons. The time interval
between platforms is

$$t = \frac{TQ_p}{Q} \ .$$

The time for a platform to deliver the cargo is the time in transit plus the
loading-unloading time. The time for a one-way transit is $\frac{D}{V}$, where

D = one-way transit distance (nm)

V = speed of transit (knots)

The loading-unloading time is $\frac{Q_p}{r}$, where r is the loading-unloading rate in tons/hour. The time for a roundtrip is

$$2 \left(\frac{D}{V} + \frac{Q_p}{r}\right) .$$

The roundtrip time divided by the time interval between platforms equals the number of platforms required to fill the pipeline. Thus, the number of platforms, n, required to fill the pipeline is given by the expression

$$n = \frac{2}{t} \left(\frac{D}{V} + \frac{Q_p}{r}\right) \qquad \text{where} \quad t = \frac{TQ_p}{Q} \qquad \text{A-13}$$

or

$$n = \frac{2Q}{T} \left(\frac{D}{VQ_p} + \frac{1}{r}\right) \qquad \text{A-14}$$

The effects of changing the loading-unloading rates on the number of platforms can be expressed in terms of the relative number of platforms required to fill the pipeline, compared to the number for a base case. This is given by

$$Y_{rel} = \frac{\left(\dfrac{D}{VQ_p} + \dfrac{1}{r}\right)}{\left(\dfrac{D}{VQ_p} + \dfrac{1}{r^*}\right)} \qquad \text{A-15}$$

where

Y_{rel} = relative number of platforms

r^* = base case loading/unloading rate

B.   CONVOY

The purpose of this section is to demonstrate the appropriate geometry derivations and equations used in the analysis of convoy operations.

1.   Area of Threat to the Convoy

The following discussion of the area of threat is an extension of Koopman's Theory of Search.*

The area of threat to the convoy at an instant in time is the area from which an attacker could detect and approach the convoy.

The area of threat is a function of the attacker speed, the attacker weapon speed, the attacker weapon range, the attacker detection range, and the convoy speed. Figure B-1 repeats Figure IV-1 and contains the appropriate geometry and relationships.

Case Ia shows the threat area when the weapon range is zero and the convoy speed is greater than the attacker speed. The limiting angle of approach is determined from relative motion considerations and is given by

$$\theta = \sin^{-1}\left(\frac{V_A}{V_C}\right),$$ B-1

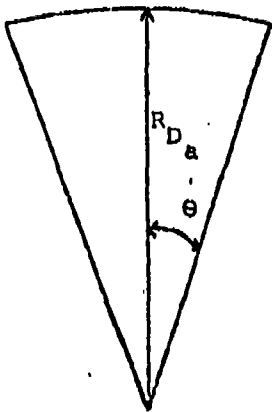where V_A = Speed of Attacker

V_C = Speed of Convoy.

The area of threat in Case Ia is the sector of a circle whose radius is equal to the attacker's detection range and whose angle is the angle between the two limiting lines of approach. This angle is (see second following page, B-3)

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B.O. Koopman, "Theory of Search, Part I, Kinematic Bases", Operations Research, Volume 4, (1956), pp 324-346.
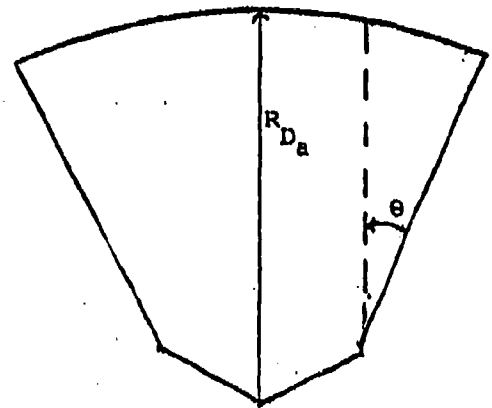
Area of a Threat to a Convoy for a Given
Attacker Detection Range As A Function of Convoy/
Attacker Speed Ratio and Convoy/Attacker Weapon Speed Ratio
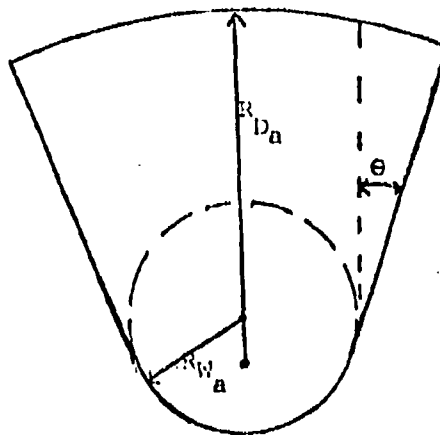and the Attacker Weapon Range

Case I $V_C > V_A$

Case Ia

$V_{W_a} = V_A$ (or $R_{W_a} = 0$)

Case Ib

$V_C > V_{W_a} > V_A$

Case Ic

$V_{W_a} > V_C > V_A$

Where: R_{D_a} = Detection Range of Attacker V_A = Speed of Attacker

R_{W_a} = Weapon Range of Attacker V_{W_a} = Speed of Attacker's Weapon

V_C = Speed of Convoy

$$20 = 2 \sin^{-1}\left(\frac{V_A}{V_C}\right).$$

Thus, the threat area is

$$A_{Ia} = R_{D_a}^2 \sin^{-1}\left(\frac{V_A}{V_C}\right)$$

and the normalized threat area for case Ia is

$$A'_{Ia} = \frac{A_{Ia}}{\Pi R_{D_a}^2},$$

where R_{D_a} = detection range of the attacker.

When the weapon speed is greater than the convoy speed, and the convoy speed is greater than the attacker speed, the threat area is as shown in Case Ic in Figure B-1.

The threat area for Case Ic is a function of the attacker speed (V_A), the convoy speed (V_C), the attacker weapon speed (V_{W_a}), the attacker weapon range (R_{W_a}) and the attacker detection range (R_{D_a}). Let point 0 (see Figure B-2) be the convoy center at the time of detection and let 0 define a circle with radius R_{D_a}.
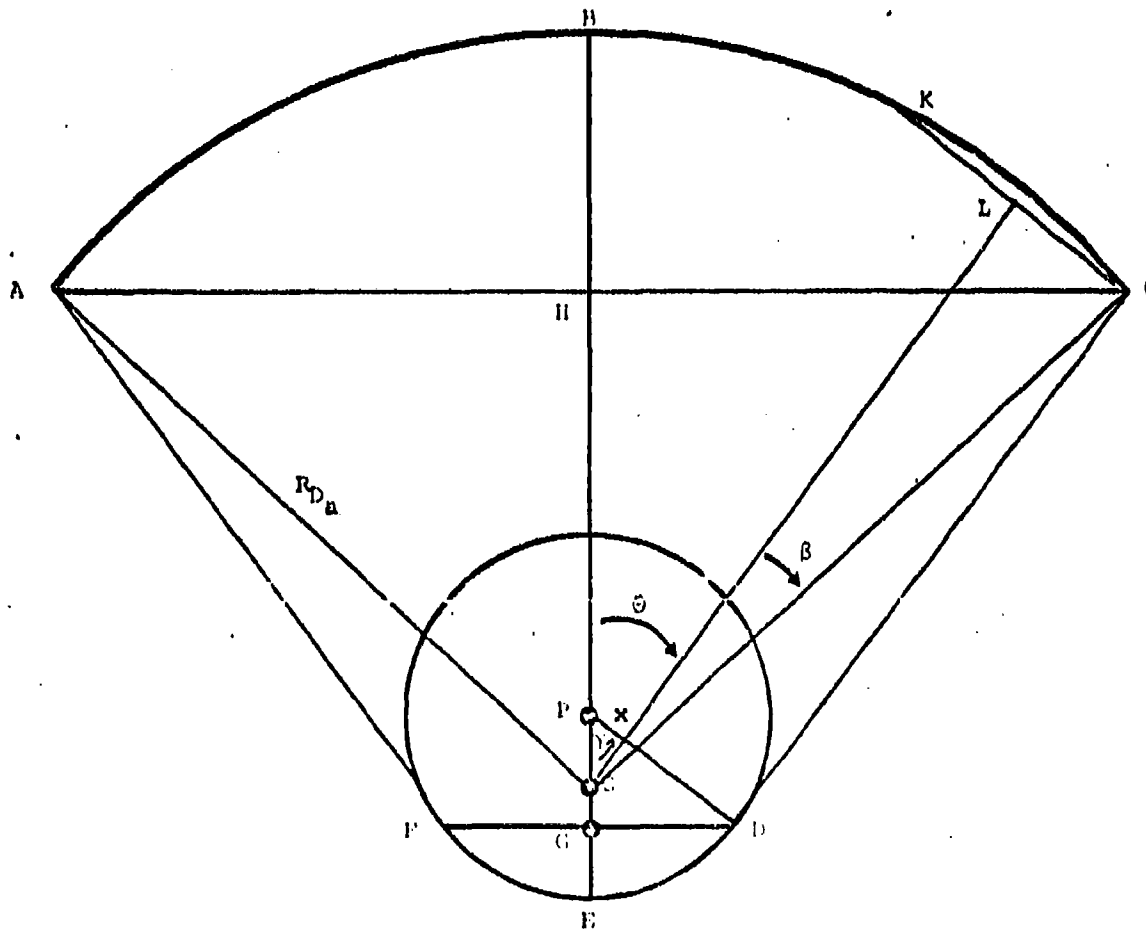
When $V_{W_a} > V_C$ and $V_C > V_A$, the instantaneous threat area is bounded by the path ABKCDEFA. (When $V_{W_a} > V_C$, the convoy can be threatened from behind by the attacker's weapons) The maximum threat distance from directly behind the convoy at point E at an instant in time is

$$R_{W_a} - \frac{V_C}{V_{W_a}} R_{W_a}.$$

When only the attacker's weapon is considered, the area of threat to the convoy as the convoy transits from 0 to P is the circle centered at P and radius R_{W_a}. Point P is a distance $\frac{V_C}{V_{W_a}} R_{W_a}$ from point 0.

Geometry for Determining the Area of Threat When Attacker Weapon Speed is Greater Than Convoy Speed and Convoy Speed is Greater Than Attacker Speed

$$\Theta = \sin^{-1} \frac{V_A}{V_C} \qquad\qquad PD = R_{W_a}$$

$$\beta = \sin^{-1}\left[\frac{R_W}{R_D}\left(1 - \frac{V_A}{V_W}\right)\right] \qquad OC = R_{D_a}$$

$$\gamma = \frac{\pi}{2} - \Theta$$

When the attacker speed is considered, the limiting lines of threat are tangent to the circle centered at P and are at an angle $\sin^{-1}\left(\frac{V_A}{V_C}\right)$ to the path of the convoy. Thus, the arc FED, the limiting lines AF and DC, and the arc ABKC are the boundaries of the instantaneous area of threat.

The instantaneous threat area can be divided into three areas--the segment FGDEF, the segment AHCKBA, and the trapezoid with top AHC, bottom FGD, and height GH.

The area is calculated as follows: the angle θ is a function of the attacker speed and convoy speed and is the limiting angle of threat for an attacker. The angle β can be calculated by determining the chord length CK. Lines OL and CD are parallel and the line CD is tangent to the circle centered at P. PD is perpendicular to CD and OL, and PD equals R_{W_a}. Chord length CK $= 2\,(R_{W_a} - x)$, where

$$x = \frac{V_C}{V_{W_a}} R_{W_a} \qquad \sin\theta = \frac{V_A}{V_{W_a}} R_{W_a} \qquad\qquad \text{B-6}$$

Thus, $\beta = \sin^{-1}\left[\dfrac{R_{W_a}}{R_{D_a}}\left(1 - \dfrac{V_A}{V_{W_a}}\right)\right]$

The segment AHCLKBA is subtended by the angle $2(\theta + \beta)$ with radius R_{D_a}. The area of the segment is,

$$\frac{\pi R_{D_a}^2\,(2\,(\theta + \beta))}{2\pi} - \frac{R_{D_a}^2}{2}\left[\sin 2\,(\theta + \beta)\right] \qquad\qquad \text{B-7}$$

The angle γ is equal to $\frac{\pi}{2} - \theta$.

So the area of the segment FEDGF is,

$$\frac{\pi R_{W_a}^2\,2\gamma}{2\pi} - \frac{R_{W_a}^2}{2}\sin(2\gamma) \qquad\qquad \text{B-8}$$

The trapezoid has a top AHC and bottom FGD. We have $AHC = 2R_{D_a} \sin(\theta + \beta)$, and $FGD = 2R_{W_a} \sin(\gamma)$.

The height of the trapezoid can be calculated as $HO + PG - OP$, where,

$$HO = R_{D_a} \cos(\theta + \beta)$$

$$PG = R_{W_a} \cos(\gamma)$$

$$OP = \frac{V_C}{V_{W_a}} R_{W_a}$$

Thus, the area of threat is given by,

$$A_{Ic} = R_{D_a}^2 (\theta + \beta) - \frac{R_{D_a}^2}{2} \sin[2(\theta + \beta)] + R_{W_a}^2 (\gamma) - \frac{R_{W_a}^2}{2} \sin(2\gamma)$$

$$+ [R_{D_a} \sin(\theta + \beta) + R_{W_a} \sin(\gamma)] \times [R_{D_a} \cos(\theta + \beta) + R_{W_a} \cos(\gamma) - \frac{V_C}{V_{W_a}} R_{W_a}] \qquad \text{B-9}$$

$$= R_{D_a}^2 (\theta + \beta) + R_{W_a}^2 (\gamma) + R_{D_a} R_{W_a} \cos(\beta)$$

$$- \frac{V_C}{V_{W_a}} R_{W_a} \left[R_{D_a} \sin(\theta + \beta) + R_{W_a} \sin(\gamma) \right]$$

when $V_{W_a} > V_C > V_A$

with

$$\theta = \sin^{-1} \left(\frac{V_A}{V_C} \right)$$

$$\beta = \sin^{-1} \left[\frac{R_{W_a}}{R_{D_a}} \left(1 - \frac{V_A}{V_{W_a}} \right) \right]$$

$$\gamma = \frac{\pi}{2} - \theta$$

and the normalized threat area for Case Ic is given by

$$A'_{Ic} = \frac{A_{Ic}}{\pi R_{D_a}^2} . \qquad \text{B-9a}$$

2. Number of Escorts Required

The section develops the requirements for the number of escorts to provide timely prosecution around the entire circumference of the threat circle discussed in the text (i.e., Case II where the attacker's speed is greater than the convoy speed.) For purposes of illuminating the problem, the convoy was considered stationary. Relative motion of the attacker increases with the component of the convoy speed toward the attacker. The effect on the required speed of the escort depends on the relative postions. In the most demanding case (attacker dead ahead of convoy, escort dead astern), the escort speed requirement is increased by an amount equal to the convoy speed.

The basic geometry is shown in Figure B-3. The escort is stationed at a point on a circle of radius R_E, about the center of the convoy. An attacker is detected at a distance R_{D_C} from the center of the convoy. The detection could be made by the escorts, the convoy, or some external source such as aircraft or satellite. Thus, the escorts could have the sole task of intercepting the attacker. The escort must then cover the distance R between his position relative to the center of the convoy and the point at which the attacker could launch his weapon in order to intercept the attack. The intercept distance, R_I, again measured from the center of the convoy, is taken to be equal to or greater than the range of the attacker's weapon. The time within which the escort must travel from the point of initial detection of the attacker to the point of interception is

$$t = \frac{R_{D_C} - R_I}{V_A} \qquad \qquad \text{B-10}$$

and (assuming an escort weapon with infinite speed) the distance to the point of interception is

$$R = V_E t + R_{W_\theta} \qquad \qquad \text{B-11}$$

B-7

Where V_A = Speed of the attacker

V_E = Speed of the escort

R_{W_e} = Range of the escorts weapon

The angle, ω, shown in Figure B-3, is one half the sector coverage of a single escort and is given by

$$\omega = \cos^{-1}\left[\frac{R_I^2 + R_E^2 - R^2}{2\,R_I R_E}\right] \qquad \text{B-12}$$

Then the number of escorts required is

$$N = \frac{\pi}{\omega} \qquad \text{B-13}$$

Since the number of escorts required is a function of the distance that each escort is stationed from the center of the convoy, R_E, the number of escorts can be minimized for given values of R and R_I by maximizing ω with respect to R_E.

Setting
$$\frac{d\omega}{dR_E} = 0$$

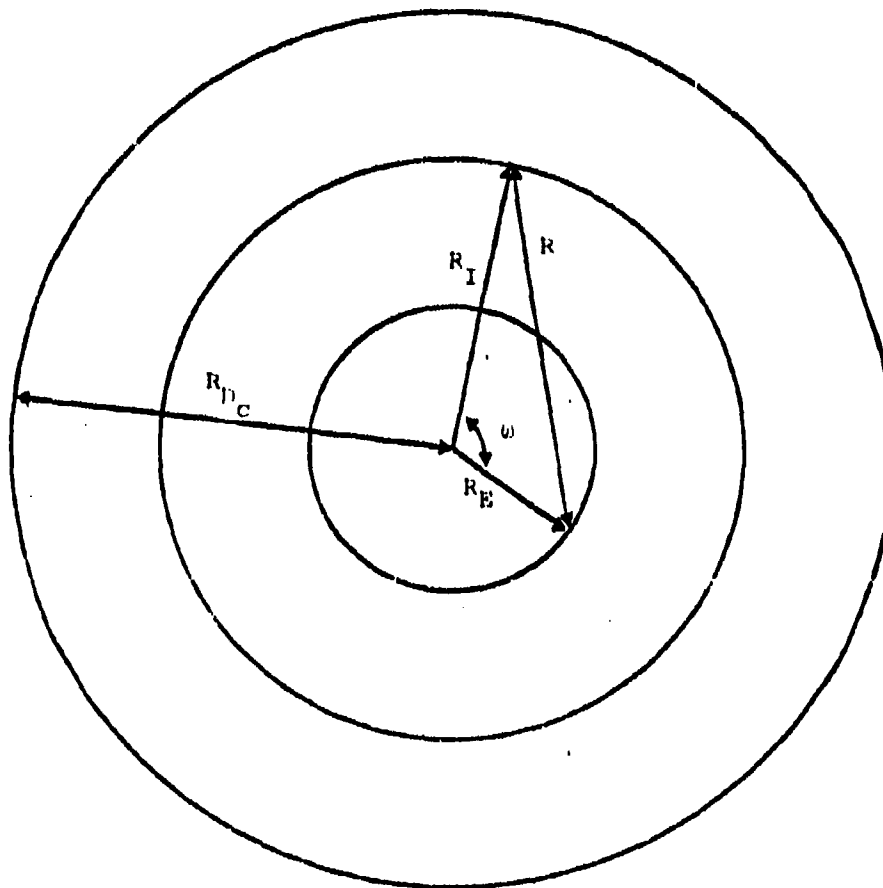and after simplifying, we get,

$$R_I^2 = R_E^2 + R^2$$

which is recognized as a right triangle with sides R_E and R.

ω_{max} then becomes,

$$\omega_{max} = \sin^{-1}\left(\frac{R}{R_I}\right) = \sin^{-1}\left[\left(\frac{V_E}{V_A}\right)\left(\frac{R_{D_c} - R_I}{R_I}\right) + \frac{R_{W_e}}{R_I}\right] \qquad \text{B-14}$$

Figure B-3

Geometry for Determining the
Sector Coverage of a Single Escort

R_E = Escort Distance*

R_I = Intercept Distance*

R_{D_c} = Detection Distance*

ω = Sector Half Angle

_____

*All distances are measured from the center of the convoy.

and the minimum number of escorts required is now

$$N_{min} = \frac{\pi}{\omega_{max}}$$

B-15

Considering the integer constraint, the minimum number of escorts is ≥ 2 whenever $R > R_I$. At $R = R_I$ a single escort, stationed in the center of the convoy, suffices.

3. Escort Sprint Speed Requirements

This section develops the escort sprint speed required for a given convoy speed of advance. The escort must maintain a speed of advance equal to or greater than the convoy speed of advance, V_c. The escort's speed of advance is determined by his sprint speed, V_E, the time he spends drifting, T_D and his acoustic detection range, R_{D_c}.

The geometry is shown below.
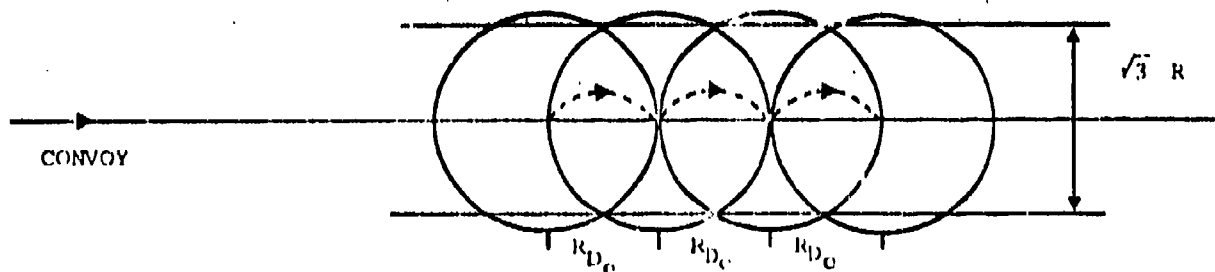
CONVOY

In this case, the sprint distance is equal to R_{D_c}. By choosing this separation, the escort sweeps a width equal to $\sqrt{3}R$, normal to the convoy's speed vector, providing convoy and escort maintain a constant course.

The time required for the convoy to travel one sprint distance is simply,

$$T_c = \frac{R_{D_c}}{V_c} \quad ,$$

B-16

and the time available for the escort to maintain station while covering an equivalent distance is,

$$T_E = \frac{R_{D_C}}{V_E} + T_D \qquad\qquad \text{B-17}$$

In order for the escort to maintain a speed of advance equal to the convoy speed of advance. Then, since

$$T_E = T_C$$

we get

$$\frac{R_{D_e}}{V_E} + T_D = \frac{R_{D_c}}{V_C} \qquad \text{or,} \qquad \text{B-18}$$

$$V_E = \frac{V_V V_C}{V_V - V_C} \qquad\qquad \text{B-19}$$

where

$$V_V = \frac{R_{D_e}}{T_D}$$

If we increase the number of escorts to n, then the sprint distance becomes nR_{D_c} and equation B-19 becomes

$$V_E = \frac{nR_{D_c} V_C}{nR_{D_c} - V_C T_D}$$

$$= \frac{V_V V_C}{V_V - V_C} \qquad\qquad \text{B-20}$$

where V_V has been generalized to

$$V_V = \frac{nR_{D_c}}{T_D} = \frac{\text{sprint distance}}{\text{drift time}} = \text{virtual speed.}$$

The figure below shows the case n = 2, with escorts A and B "leap-frogging" along
the path of the convoy.
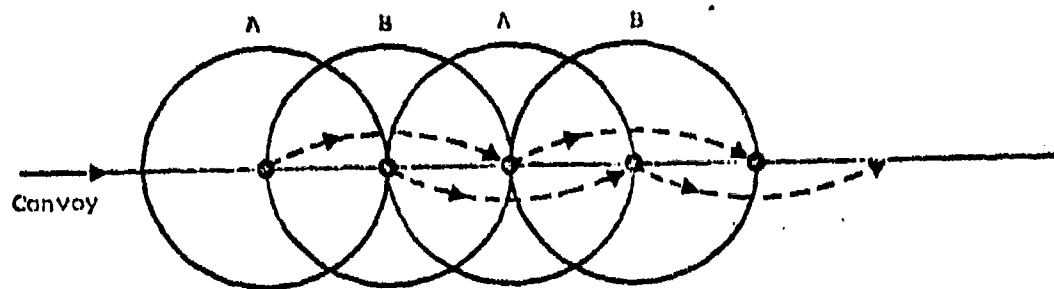
C. SEARCH

1. General

This section of the Appendix provides geometry and functional relationships for the analysis of the effects of speed on search operations in the text. Discussion is limited to acoustic search and to reiteration of general functional relationships in acoustics and acoustic search which are pertinent to an investigation of the utility of search vehicle speeds.

There are two important factors which tend to bound the speed range of interest for acoustic search. For surface or near surface platforms, flow noises at speeds in excess of 30 knots reach a level at which the detection range is for all practical purposes, zero. Herculean design efforts appear to be necessary to produce any increase in this limiting speed.

At very low speeds, the prevailing background noise in the sea dominates the problem. Thus, the theoretical detection ranges which might be achieved in a noiseless environment do not occur in the real world. In general, detection ranges are limited by the environment to a constant value until searcher speed reaches about 10-15 knots, and then decrease with increasing speed, reaching the zero value at about 30 knots.

Thus, the search speed of interest, for the foreseeable future, lies between 10-15 knots and about 30 knots. This suggests that the projected speed capabilities of most of the advanced naval vehicle concepts (with the possible exception of SWATH ships) gain little or no support from search function. This is not entirely true since, in the analysis of sprint-drift or flying-drift search, we find a clear case for high sprinting (or flying) speeds between search periods.

2. Barrier Search

The purpose of this section is to investigate the impact of search speed on the probability of detecting a submarine transiting a barrier. The geometry of the problem is shown below:
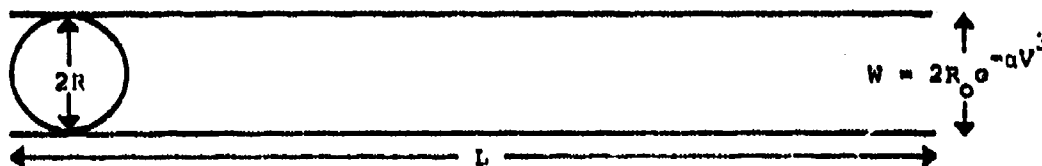
a. Continuous Search

Initially, we assume a searcher conducting a continuous random search in his barrier station. Both active and passive sonar search are addressed. For both methods, the detection range is degraded with speed due to flow noise considerations.

Over the speed range of interest (10-30 knots) detection range as a function of searcher speed (V) is approximated by:

$$R = R_o e^{-aV^3} \qquad\qquad C-1$$

where R_o = maximum detection range for the given conditions (generally, range at speeds of 0-10 knots)

$a = 3 \times 10^{-4}$

V = search speed (knots)

and the equation closely approximates empirical data which indicates flow noise increasing at a nearly linear rate of 1.8db/knot over the speed range of 10-20 knots.*

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*R.J. Urick, Principles of Underwater Sound for Engineers, (New York, 1967).

The sweep rate is then given by:

$$\text{Sweep Rate} = 2RV = 2R_o V e^{-\alpha V^3} \left(\frac{nm^2}{hr}\right) \qquad \text{C-2}$$

The product of the sweep rate and some time (t) defines a rectangle of width $2R_o e^{-\alpha V^3}$ and length Vt to which one adds the end semicircles, with total area equal to $\pi (R_o e^{-\alpha V^3})^2$, to obtain area swept in t, which is

$$\text{Area Swept} = 2R_o e^{-\alpha V^3} Vt + \pi (R_o e^{-\alpha V^3})^2 \qquad \text{C-3}$$

The time required for the submarine to transit the barrier is given by

$$t = \frac{W}{V_s} \ (hr)$$

where V_s = submarine speed (knots)

 W = width of throat of detection to the target (nm)

The total area swept by the searcher in time, t, is then:

$$\text{Total Area Swept} = \frac{2R_o e^{-\alpha V^3} VW}{V_s} + \pi (R_o e^{-\alpha V^3})^2 \qquad \text{C-4}$$

and since

$$W = 2R_o e^{-\alpha V^3} \ (nm) \ ,$$

$$\text{Total Area Swept} = \frac{4V(R_o e^{-\alpha V^3})^2}{V_s} + \pi (R_o e^{-\alpha V^3})^2$$

The standard expression for the probability of detecting a submarine transiting a barrier is:

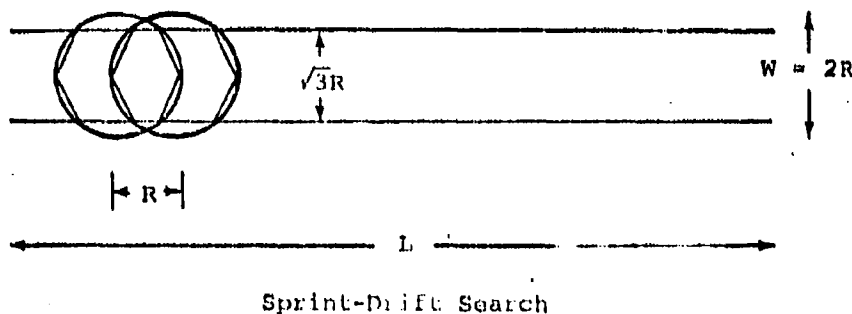$$P_D = 1 - e^{-\frac{\text{Total Area Swept}}{\text{Barrier Area}}} \qquad \text{C-5}$$

Therefore,

$$P_D = 1 - e^{-\left[\frac{2VR_o e^{-\alpha V^3}}{LV_s} + \frac{\pi (R_o e^{-\alpha V^3})}{2L}\right]} \qquad \text{C-6}$$

where L = length of barrier.

C-3

b.   Sprint-Drift Search

The purpose of this section is to investigate the impact of search speed on the probability of detecting a submarine transiting a barrier by using sprint-drift search.   The geometry of the problem is shown below:

Sprint-Drift Search

The result is an overlapping search pattern wherein the searcher sprints a distance R at a speed $\bar{V}$ then drifts and listens for a time T_D.   He then sprints another distance R and continues to repeat the maneuver.   Detection range is equal to R (maximum for the environment) since searching is confined to the drifting period.

The time to complete one segment of the sprint-drift search is:

$$T = \frac{R}{\bar{V}} + T_D \quad (hr)$$

C-7

Where   R = detection range (nm)

$\bar{V}$ = sprint speed (knots)

T_D = drift (listen) time (hr)

The speed of advance of the searcher is then given by:

$$V' = \frac{R\bar{V}}{R + \bar{V}T_D} \quad (knots) \qquad \text{C-8}$$

The sweep rate then becomes:

$$\text{Sweep Rate} = \sqrt{3}\ RV' = \frac{\sqrt{3}R^2\bar{V}}{R + \bar{V}T_D} \qquad \left(\frac{nm^2}{hr}\right) \qquad \text{C-9}$$

The time, t, required for a submarine to transit the barrier is

$$t = \frac{W}{V_s}$$

where W = width of barrier (nm)

V_s = speed of submarine (knots)

Therefore, the total area swept out in time, t, using sprint-drift tactics is

$$\text{Total Area Swept} = \frac{\bar{V}\sqrt{3}R^2 W}{V_S(R + \bar{V}T_D)} \qquad (nm^2) \qquad \text{C-10}$$

When the initial area of detection, πR^2, is added to the total area swept and the barrier width is 2R we have:

$$P_D = 1 - e^{-\left[\frac{\bar{V}\sqrt{3}R^2}{V_S L(R + \bar{V}T_D)} + \frac{\pi R}{2L}\right]} \qquad \text{C-11}$$

Alternative search tactics exist. Section F of this appendix compares the overlapping search tactic above with a random, non-overlapping search.

3. Open Area Search

The purpose of this section is to investigate the impact of search speed on the expected number of targets detected per hour using continuous open area search. The detection range is degraded with speed due to flow noise as in the barrier case. The targets have an average density per square nautical mile and have uniformly distributed track angles.

The geometry of the problem is shown below.
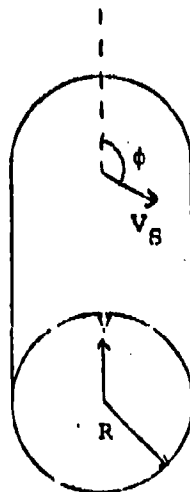
Open Area Search

a. Continuous Search

The expression for the number of targets detected per hour is based on Koopman's theory of search, and is modified in this analysis to include the effect of flow noise on detection range as the search speed is increased.

The number of targets detected per hour is given by.

$$N_o = (V + V_B) \frac{4N}{\pi} R_o e^{-\alpha V^3} \int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{4VV_B}{(V + V_B)^2} \sin^2 \psi} \, d\psi \qquad \text{C-12}$$

C-6

Where $V =$ search speed   (knots)

$V_s =$ target speed   (knots)

$N =$ target density (number per nm^2)

$R_D =$ detection range at zero speed (nm)

$\alpha = 3 \times 10^{-4}$

$\psi = (\pi - \phi)/2$

$\phi =$ target track angle (degrees)

The integral expression is an elliptic integral and is readily evaluated using standard tables of elliptic integrals.

b. Sprint-Drift Search

The purpose of this section is to investigate the impact of search speed on the expected number of targets detected per hour using sprint-drift tactics. The geometry is essentially the same as in the barrier case, with the exception that the search area is unbounded.

A first order approximation to the expected number of targets to pass within a distance, R, of the searcher during a drift (listen) period is given by:

$$N_o = N\pi R^2 + 2NRV_s T_D \qquad \text{C-13}$$

where the quantities in the expression have been previously defined.

The number of targets detected during one listen period is the sum of the targets inside the radius of detection at the beginning of the period plus the number that enter during the listen period.

Hence, the expected number of targets detected per hour is the number detected per listen period divided by the duration of the cycle, i.e.,

$$N_o = \frac{N\pi R^2 + 2NRV_s T_D}{\frac{R}{\bar{V}} + T_D} \qquad \text{C-14}$$

The number of targets detected per hour given in equation C-14 illustrates the impact of speed when sprint-drift search is used. This approximation can be modified to consider different approaches to the search tactic. For example, equation C-14 does not differentiate between targets that were detected on previous looks and targets that are new detections. Thus, some targets are counted more than once. These duplicated detections could be subtracted to give the number of new detections per hour. The sprint distance could be optimized for a given sprint speed. In addition the number of detections at zero speed depends on the initial assumptions about the searcher. For example, if the search is required to sprint a given distance before listening, then the number of detections at zero speed is zero.

D. PURSUIT

The purpose of this appendix is to demonstrate the appropriate geometry, derivations and equations used in the analysis of the utility of speed in pursuit.

1. Pursuit Curve*

The geometry for deriving the curve of pursuit is shown in Figure D-1.

The curve (AB) is traced by a point P (pursuer) which moves in such a manner that its direction of motion is always pointed toward a second point P' (pursuee) which moves along the path (CD). The speeds of P and P' are taken to be constant. The problem proposed is to construct the curve of pursuit (AB) when (CD) is given and the speeds of P and P' are known.

The simplest problem of this type is that for which the path of the pursuee is a straight line.

Let $P = (x,y)$ be a point on the pursuer's curve and $P' = (w,z)$ be a point on the path of the pursuee.

The curve traced by P' is given by

$$f(w,z) = 0 \qquad\qquad\qquad \text{D-1}$$

Since the tangent through P passes through P' the pursuit equation can be written as

$$(z - y) = \frac{dy}{dx} (w - x) \qquad\qquad \text{D-2}$$

Let the speed ratio of the pursuer to pursuee be given by ζ, then

---

*Introduction to Nonlinear Differential and Integral Equations, U. S. Atomic Energy Commission, U.S. Government Printing Office, (Washington, D.C., 1960) pp 113-128.

Figure D-1

Geometry For Deriving The Curve Of Pursuit
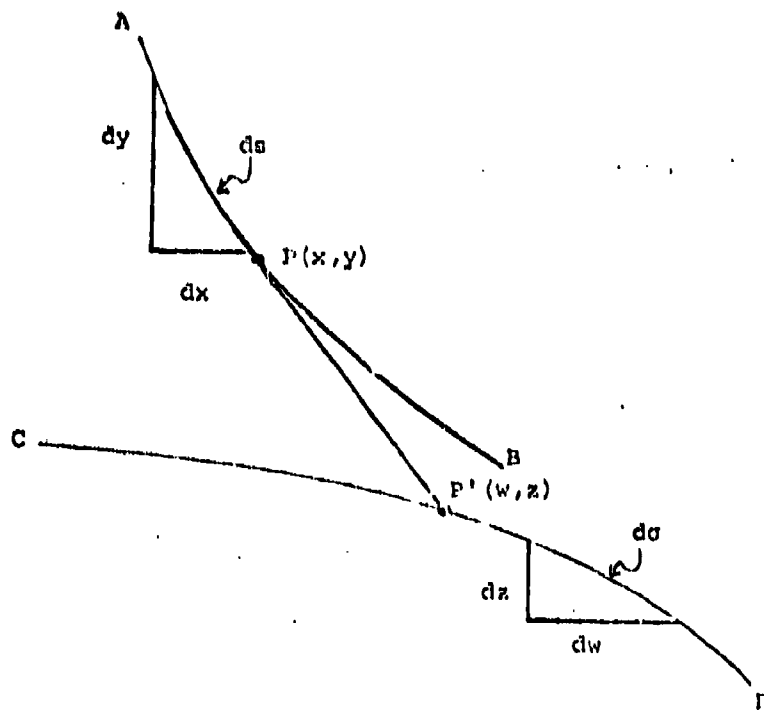
P = Pursuer
P' = Pursuee
AB = Path of Pursuer
CD = Path of Pursuee
ds = Element of Arc on Pursuer's Path
dσ = Element of Arc on Pursuee's Path

$\frac{ds}{dt} = \zeta \frac{d\sigma}{dt}$, where ds and $d\sigma$ are the elements of the arcs of the pursuer and pursuee respectively. We then have the equation

$$dx^2 + dy^2 = \zeta^2 \ (dw^2 + dz^2) \qquad\qquad D\text{-}3$$

Since y, w and z are functions of x, D-3 can be written in the form

$$1 + \left(\frac{dy}{dx}\right)^2 = \zeta^2 \left[\left(\frac{dw}{dx}\right)^2 + \left(\frac{dz}{dx}\right)^2\right] \qquad\qquad D\text{-}4$$

Differentiating D-1 and D-2 with respect to x, we get

$$\frac{\partial f}{\partial w} \frac{dw}{dx} + \frac{\partial f}{\partial z} \frac{dz}{dx} = 0 \qquad\qquad D\text{-}5$$

and

$$\frac{dz}{dx} = \frac{d^2y}{dx^2} \ (w - x) + \frac{dy}{dx} \frac{dw}{dx} \qquad\qquad D\text{-}6$$

When the proper values for D-1, D-2, D-5 and D-6 are substituted into the right-hand member of equation D-4, the differential equation of the curve of pursuit is obtained.

Applying this general theory to determine the curve of pursuit, when the pursuee moves along a straight line parallel to the y-axis and a distance d from the origin, we get

$$w = d \quad \text{and} \quad \frac{dw}{dx} = 0.$$

From equation D-6,

$$\frac{dy}{dx} = \frac{d^2y}{dx^2} \ (d - x) \qquad\qquad D\text{-}7$$

Substituting D-6 into D-4, the following differential equation is obtained.

$$1 + \left(\frac{dy}{dx}\right)^2 = \zeta^2 (d - x)^2 \left(\frac{d^2y}{dx^2}\right)^2 \qquad\qquad D\text{-}8$$

Integrating D-8 twice yields the equation

$$y = \frac{1}{2}\left[\frac{c}{1-\zeta}(d-x)^{1-1/\zeta} + \frac{\zeta}{c(1+\zeta)}(d-x)^{1+1/\zeta}\right] + c'$$

D-9

Figure D-2

Geometry For Pursuit Curve When
Pursuee Moves Along a Straight Line
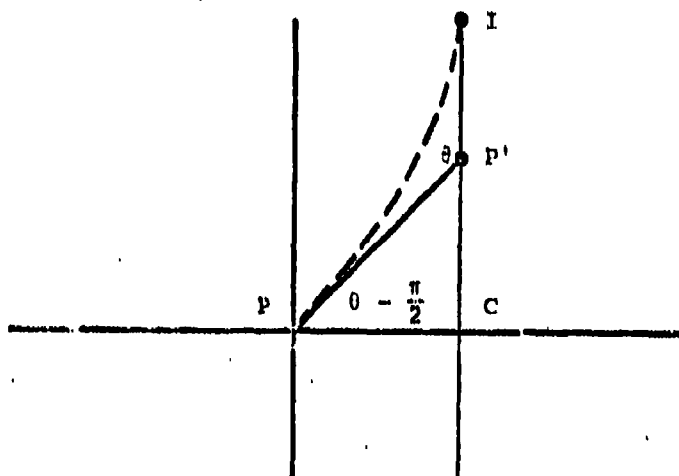
$PC = d$

$PP' = d' = d \sec(\theta - \frac{\pi}{2}) = d \csc\theta$

$PI = $ Path of Pursuer

$P'I = $ Path of Pursuee

At $x = 0$, $y = 0$, and $\frac{dy}{dx} = \tan(\theta - \frac{\pi}{2}) = -\cot\theta$. Now,

$$\frac{dy}{dx} = \frac{1}{2}\left[c(d-x)^{-1/\zeta} - \frac{1}{c}(d-x)^{1/\zeta}\right]$$

D-10

D-4

At $x = 0$

$$-\cot\theta = \frac{1}{2}\left[cd^{-1/\zeta} - \frac{1}{c}\,d^{1/\zeta}\right]$$

$$c = d^{1/\zeta}(-\cot\theta \pm \csc\theta) \qquad\qquad \text{D-11}$$

We reject $-d^{1/\zeta}(\cot\theta + \csc\theta)$ as a spurious solution (it gives the case when the direction of the pursuee is reversed, i.e., when the tracking angle is $\pi - \theta$).

Substitution for c in equation D-9 yields (at $x = 0$)

$$0 = \frac{1}{2}\left[\frac{\zeta}{1-\zeta}(-\cot\theta + \csc\theta)d + \frac{\zeta}{1+\zeta}\left(\frac{1}{-\cot\theta + \csc\theta}\right)d\right] + c' \qquad\qquad \text{D-12}$$

Solving for c'

$$c' = \left(\frac{\zeta\csc\theta - \zeta^2\cot\theta}{\zeta^2 - 1}\right)d \qquad\qquad \text{D-13}$$

If $\zeta > 1$, capture takes place when $x = d$, i.e., when

$$y = \left(\frac{\zeta\csc\theta - \zeta^2\cot\theta}{\zeta^2 - 1}\right)d$$

$$= \left(\frac{\zeta - \zeta^2\cos\theta}{\zeta^2 - 1}\right)d\csc\theta$$

$$= \left(\frac{\zeta - \zeta^2\cos\theta}{\zeta^2 - 1}\right)d' = IC \qquad\qquad \text{D-14}$$

The capture distance is thus,

$$IP' = IC - P'C$$

$$= \left(\frac{\zeta - \zeta^2\cos\theta}{\zeta^2 - 1}\right) d' - d'\sin(\theta - \frac{\pi}{2})$$

$$= \left(\frac{\zeta - \zeta^2\cos\theta}{\zeta^2 - 1} + \cos\theta\right) d'$$

$$= \left(\frac{\zeta - \cos\theta}{\zeta^2 - 1}\right) d' \qquad\qquad\qquad\qquad \text{D-15}$$

where d' is the initial separation distance.

The derivations above take the weapon range, R_w, to be 0. If we consider a pursuer with a positive weapon range, R_w, and infinite weapon velocity, then capture occurs when the distance between pursuer and pursuee is R_w. In the following derivation we take the tracking angle to be $\frac{\pi}{2}(= 90^0)$ and normalize all distances to the initial separation distance d.

Figure D-3
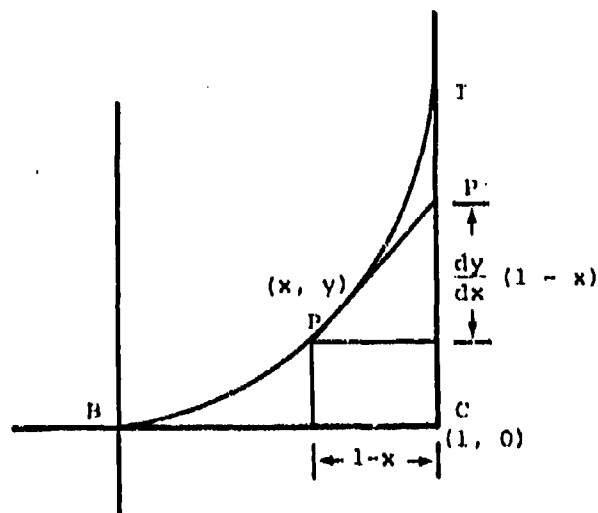
BC = normalized initial separation distance

PP' = normalized distance between pursuer and pursuee

BPI = pursuer's path

CP'I = pursuee's path

$$\frac{R_W}{d} = PP'$$

$$= \sqrt{\left[\frac{dy}{dx}(1-x)\right]^2 + (1-x)^2} \qquad \text{D-16}$$

$$= \sqrt{\left(\frac{dy}{dx}\right)^2 + 1}\,(1-x)$$

$$= \sqrt{\left[(1-x)^{-1/\zeta} + (1-x)^{-1/\zeta}\right]^2}\,(1-x)$$

$$= (1-x)^{1-1/\zeta} + (1-x)^{1+1/\zeta} \qquad \text{D-17}$$

If we let $z = (1-x)^{1/\zeta}$, D-17 becomes

$$z^{\zeta+1} + z^{\zeta-1} - \frac{R_W}{d} = 0 \qquad \text{D-18}$$

For arbitrary ζ, D-18 does not have a general solution so we must use numerical methods, e.g., Newton's method of iteration, to evaluate z.

   The normalized capture distance is thus

$$P'C = y + \frac{dy}{dx}(1-x)$$

$$= \frac{\zeta}{\zeta^2-1} - \frac{1}{2}\left[\frac{1}{\zeta-1}z^{\zeta-1} + \frac{1}{\zeta+1}z^{\zeta+1}\right]. \qquad \text{D-19}$$

The capture distance is $(d)(P'C)$.

2.   Constant Bearing Intercept

The geometry for deriving the equation for constant bearing intercept is shown in Figure D-4.

The line (OI) is the projected track of the pursuee and the line (QI) is the intercept course followed by the pursuer.

The pursuee is initially detected at O, a separation distance, d, from the pursuer, who is located at Q. The pursuee's direction is at an angle θ to the direction OQ.

Figure D-4

Geometry For Constant Bearing Intercept
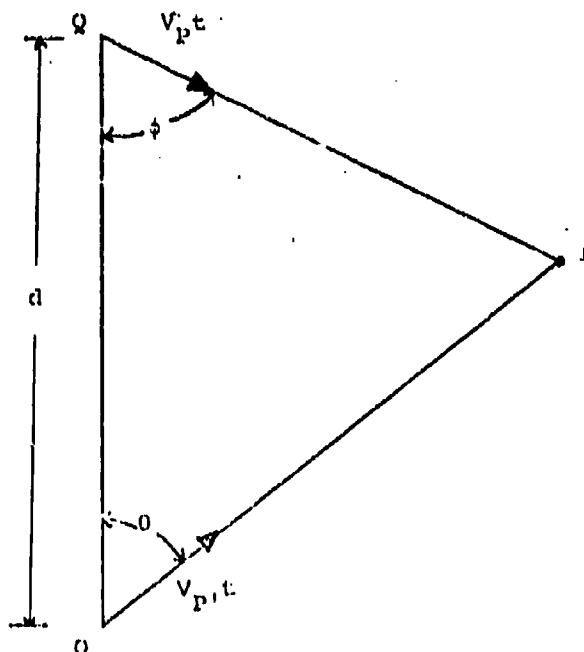
OQ = d = Initial Separation Distance
OI = Projected Track of Pursuee
QI = Intercept Path of Pursuer
θ = Initial Track Angle
ϕ = Pursuer's Lead Angle
V_p' = Pursuee's Speed
V_p = Pursuer's Speed
t = Time

D-8

The pursuee then moves along his projected track (OI) at a constant speed, $V_{p'}$. The pursuer sets off on his course (QI) and lead angle ϕ, to intercept the pursuee at point I. The pursuer also maintains a constant speed V_p.

By the law of cosines

$$d^2 + (V_{p'}t)^2 - 2dV_{p'}t\cos\theta - (V_p t)^2 = 0 \qquad \text{D-20}$$

$$d = \frac{2V_{p'}t\cos\theta \pm \sqrt{4(V_{p'}t)^2\cos^2\theta - 4\left[(V_{p'}t)^2 - (V_p t)^2\right]}}{2}$$

or $d = V_{p'}t\cos\theta \pm \sqrt{(V_{p'}t)^2(\cos^2\theta - 1) + (V_p t)^2}$ \qquad D-21

Dividing through by $V_{p'}t$, and letting

$$\zeta = \left(\frac{V_p}{V_{p'}}\right)$$

then

$$\frac{d}{V_{p'}t} = \cos\theta \pm \sqrt{\zeta^2 + (\cos^2\theta - 1)} \qquad \text{D-22}$$

Since the desired measure is capture distance/initial separation distance, we invert D-13 and obtain

$$\frac{V_{p'}t}{d} = \left[\cos\theta \pm \sqrt{\zeta^2 + (\cos^2\theta - 1)}\right]^{-1} \qquad \text{D-23}$$

Suppose that the pursuer is armed with a weapon that has a range R_W. The pursuer's objective is to maintain a constant (steady) bearing course and come within a distance R_W of the pursuee in a specified time, t. Then by the law of the cosines:

$$(V_p t)^2 = (d - R_W)^2 + (V_p' t)^2 - 2(d - R_W)(V_p' t) \cos\theta \qquad \text{D-24}$$

or

$$d - R_W = \frac{2 V_p' t \cos\theta + \sqrt{4(V_p' t)^2 \cos^2\theta - 4\left[(V_p' t)^2 - (V_p t)^2\right]}}{2} \qquad \text{D-25}$$

The weapon range required is

$$R_W = d - \left(\cos\theta + \sqrt{\cos^2\theta - 1 + \zeta^2}\right) V_p' t. \qquad \text{D-26}$$

The pursuer could improve his performance by heading on a course so that he would travel the minimum possible distance and still be a distant R_W from the target in a specified time, t. This is given by

$$(V_p t + R_W)^2 = d^2 + (V_p' t)^2 - 2d V_p' t \cos\theta$$

or

$$R_W = \left(\sqrt{d^2 + (V_p' t)^2 - 2d V_p' t \cos\theta}\right) - V_p t. \qquad \text{D-27}$$

This section derives the basis for determining the probability that a pursuer is able to relocate a pursuee with the pursuer's on-board sensor system at some time after the pursuer has received information that the pursuee is a distance d away. The information processing and data link time is taken to be zero. The pursuer travels on a straight course through the last known position of the pursuee. The pursuee is assumed to be a point target (and not an area target, such as a wake) and to move in any direction within the area of uncertainty. This area is the circle which encloses the area of possible target location. Its radius is equal to the sum of the initial location error plus the product of the pursuee's speed and the elapsed time since this location was made. In this simple case it is assumed that the pursuer's sensor has a swath width W_s within which the probability of detection is one. Outside this band the probability of detection is zero.
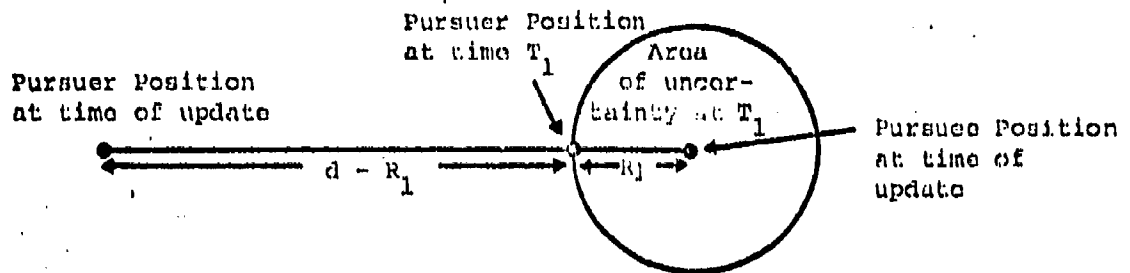
When the location error of the intermittent information system is negligible, the initial area of uncertainty that could contain both the pursuer and the pursuee has a radius

$$R_1 = V_{p'} T_1 \qquad\qquad\qquad\qquad \text{D-28}$$

where $V_{p'}$ = pursuee speed and T_1 = time for pursuer to transit $d - R_1$. (See Figure D-5)

Figure D-5

Pursuer Position
at time of update

Pursuer Position
at time T_1

Area
of uncer-
tainty at T_1

Pursuee Position
at time of
update

$d - R_1$ R_1

The pursuer searches on a constant course, with a constant swath width, through
the area of uncertainty. As the pursuer transits through the area of uncer-
tainty, the area of uncertainty increases as a function of the pursuee's speed
and the time for the pursuer to transit through the area of uncertainty.

The incremental change in the probability of detection is

$$\Delta P_D = (1 - P_D) \frac{W_S V \Delta T}{\pi (R_1 + V_p \cdot T)^2}$$

D-29

where

$1 - P_D$ = probability the target was not detected in previous ΔT's

W_S = swath width of pursuer

V = mean relative speed of pursuer and pursuee

$W_S \Delta T$ = area swept in ΔT

$\pi(R_1 + V_p, T)^2$ = area of uncertainty as it increases with time

T = time that elapses after pursuer reaches R_1

Integrating

$$P_D = 1 - \exp\left[-\frac{W_S VT}{\pi R_1 (R_1 + V_p, T)}\right] \qquad \text{D-30}$$

Since V = mean relative speed of the pursuer (V_p) and the pursuee ($V_{p'}$), we have

$$V = \frac{2}{\pi}(V_p + V_{p'}) \int_0^{\frac{\pi}{2}} \sqrt{1 - \sin^2\sigma \sin^2\psi} \; d\psi$$

$$= \frac{2}{\pi}(V_p + V_{p'}) E(\sigma)$$

$$\text{where,} \quad \sin\sigma = \frac{2\sqrt{V_p V_{p'}}}{V_p + V_{p'}} \qquad \text{D-31}$$

$$\text{and,} \quad \psi = \frac{\pi - \phi}{2}$$

E (σ) is an elliptic integral of the second kind, readily evaluated using standard tables of elliptic integrals.

The time for the pursuer to complete a first pass through the area of uncertainty is

$$T_2 - T_1 = \frac{2d V_{p'}}{V_p^2 - V_{p'}^2} \qquad \text{D-32}$$

where T_2 = time for pursuer to overtake pursuee if the track angle were 180° and T_1 = time for pursuer to overtake pursuee if track angle were 0°.

The probability that the pursuer detects the pursuee on the first pass through the area of uncertainty is

$$P_D = 1 - \exp\left[-\frac{4 \, W_S E(\sigma)\left(1 + \frac{V_p}{V_{p'}}\right)}{\pi^2 d}\right] \qquad \text{D-33}$$

4. Sprint-Drift Pursuit with Intermittent Information

A special case of pursuit with intermittent information is one where the pursuer must resort to a sprint-drift pursuit tactic. A typical case would be one where the pursuee is a high speed submarine and the pursuer's sensor is acoustic and will not function continuously at the high speeds required.

In this analysis the pursuer always sprints to the last known position of the submarine. This is due to the fact that the submarine may run on any course during the pursuer's sprint period and therefore it does not benefit the pursuer to attempt to anticipate the submarine's new course and speed. Hence this process can be viewed as a modified pursuit course since the pursuer proceeds to the last known position of the submarine, as opposed to heading toward the actual position as in the case of pure pursuit.

At some initial time, t_0, the pursuer detects a submarine at some initial distance, R_0. He then sprints to this datum at a given speed, V_p, and listens for a time, T_D.

The time elapsed during the first sprint-drift period is

$$t_1 = \frac{R_0}{V_p} + T_D \qquad\qquad \text{D-34}$$

During this time, the submarine has traveled a distance

$$R_1 = V_{p'} t_1 \qquad\qquad \text{D-35}$$

where $V_{p'}$ = submarine (pursuee) speed.

The time for the next sprint-drift period is, then,

$$t_2 = \frac{R_1}{V_p} + T_D \qquad\qquad \text{D-36}$$

and the submarine travels a distance given by $R_2 = V_{p'} t_2$.

The process is repeated until the <u>limiting value $R_n = R_{n-1}$</u> is reached.

Since the location of each drift (search) portion of the cycle is the location of the pursued at the end of the previous drift period and each drift period requires a fixed time (T_D), this limit of convergence is reached when the ground gained between sprints equals that lost while listening, i.e. if the pursuer had infinite sprint speed the limiting distance would still be given by the product of the submarine speed and the drift time.

The total time elapsed during the sprint-drift pursuit is then given by

$$T = t_1 + t_2 + t_3 + \ldots + t_n .$$

Using these basic expressions, the separation distance can be determined by an iterative process for each successive sprint-drift, and from this a separation distance history of closing distance versus elapsed time may be plotted.

E. MANEUVER AND AVOIDANCE

1. Maneuver to Avoid an Approaching Weapon

This section derives an equation for determining the speed at which a platform must maneuver in order to avoid an approaching weapon. Several restrictive assumptions are made about the weapon and the maneuvering target; thus, the derivation is illustrative and not definitive.

Figure E-1 illustrates the geometry involved for this case. The weapon and target are heading directly toward each other. The target chooses to maneuver when the weapon is a distance, D_S, from the target. The distance D_S depends on the weapon characteristics, the target characteristics and the potential escape path. The escape path used in this case is a path that is normal to the minimum radius of turn that the attacking weapon can make. The minimum radius turn that a weapon can make is a function of the weapon speed and weapon maneuverability (i.e., number of gs the weapon can pull). In this section the minimum turn radius of the weapon is defined by a radius R, as follows:

$$R = \frac{V_W^2}{ng} \qquad \text{E-1}$$

where,

V_W = weapon speed

n = limiting acceleration of the weapon (number of gs)

g = 32 feet/second^2

In this limiting case example, the target is assumed to have no such restriction and is able to turn instantaneously toward the escape path.

Geometry for Target Maneuvering to Avoid a Weapon

Distance Target
Must Travel
To Avoid
Weapon

Target

Weapon Path at Maximum Turning Rate

X

R_W

Separation
Distance
At Time
Target
Maneuvers

D_S

Target Escape Path
After Maneuver

D_W

Weapon

R_W = Radius of weapon
effectiveness

D_W = Distance weapon must
travel to intercept
target after maneuver

E-2
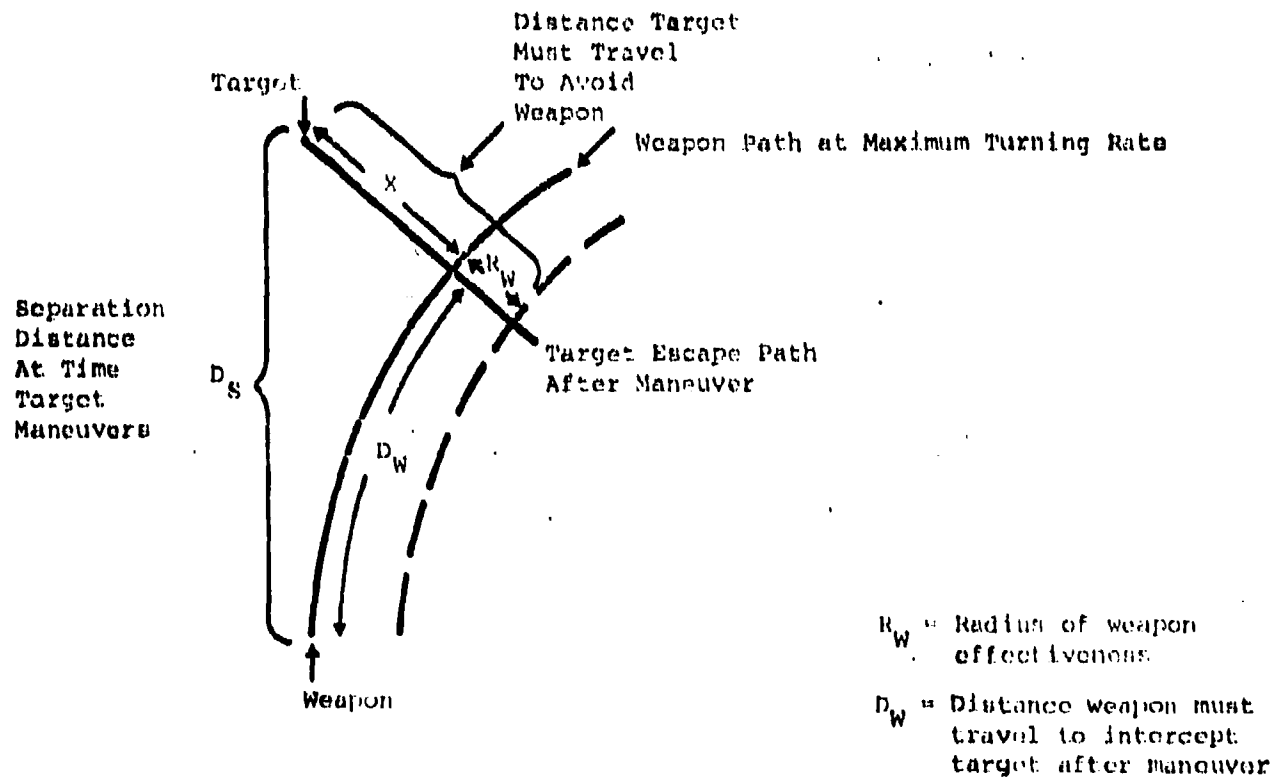
To avoid interception, the target must travel a distance $X + R_W$ before the weapon travels a distance D_W on the weapon's minimum radius turning path, where,

$$X = \sqrt{R^2 + D_S^2} - R$$

and,

D_S = separation distance between the target and the weapon when
   the target starts his maneuver

R_W = weapon lethal effect radius

$D_W = V_W T = R\Theta$

$\Theta = \tan^{-1}(\frac{D_S}{R})$

T = time for weapon to travel a distance D_W

$$= \frac{R \tan^{-1}(\frac{D_S}{R})}{V_W}$$

If $V_T T > X + R_W$, the target can escape the weapon on the target's escape path. The weapon lethal effect radius from which a target can escape by maneuver (under the previous assumptions) is given by,

$$R_W < V_T T - X. \qquad\qquad E-2$$

where V_T = target speed

If the target acts intelligently, he will begin his evasive maneuver at the time most beneficial to him, i.e, when the required weapon lethal effect radius, R_W, is maximal. Putting equation E-2 in the form where only essential parameters are present we have

$$R_W = \frac{V_T R}{V_W} \tan^{-1}\left(\frac{D_S}{R}\right) - \sqrt{D_S^2 + R^2} + R$$

Differentiating with respect to D_S and equating with zero, we obtain

$$0 = \frac{V_T R^2}{V_W} \frac{1}{R^2 + D_S^2} - \frac{D_S}{\sqrt{R^2 + D_S^2}}$$

or

$$D_S^4 + R^2 D_S^2 - \left(\frac{V_T R^2}{V_W}\right)^2 = 0$$

which yields

$$D_S = R \left(\frac{\sqrt{1 + 4\left(\frac{V_T}{V_W}\right)^2} - 1}{2}\right)^{\frac{1}{2}} \qquad E-3$$

Equation E-3 gives the optimum separation distance for a target to begin its manuever against a weapon when the target is able to turn instantaneously towards its escape path. When the value of D_S given in equation E-3 is used in equation E-2, we have the maximum weapon radius of effectiveness that can be avoided under the above assumption.

F. A COMPARISON OF TWO SPRINT-DRIFT TACTICS

This section compares two different sprint-drift (or flying-drift) tactics for search. This comparison is treated separately in the appendix, since the focus is on the relative merits of platform tactics rather than specifically on the utility of vehicle speed.

In each case:

$\bar{V}$ is the sprint (or flying) speed in knots

T_D is the drift time required to complete a search period in hours

R_D is the detection range of the sensor (at zero speed) in nautical miles

The first tactic is described in Section C of this appendix, analyzed for search effectiveness as a function of vehicle speed and used in Section V of the basic report. In the first tactic, the searcher proceeds at sprint speed for a distance R_D, stops, drifts, and searches for a period T_D. This cycle is repeated along a predetermined path of straight line segments. This tactic results in considerable overlap of search area, but there are no holes or "holidays" left unswept.

From the geometric description in Section C of the Appendix, the area swept is approximately given by:

$$A = \sqrt{3}R_D V'T \qquad\qquad F-1$$

where $\sqrt{3}R_D$ is the sweep width

V' is the overall speed of advance

with

$$\frac{1}{V'} = \frac{1}{\bar{V}} + \frac{T_D}{R_D} \qquad\qquad F-2$$

and T is the total time of search. So long as T is large (i.e., several cycles) the approximation is close to the true area swept. The principal difference

consists of the semi-circles at the end and beginning of the search and any area lost in changing the direction of the search path.

For a continuing search (where T is large) the sweep rate is the total area swept in time T. Since each sprint (flight) covers a distance R_D at a speed of $\bar{V}$, from F-1 and F-2 above, we have an overall search rate of:

$$\frac{A}{T} = \frac{\sqrt{3}R_D^2}{\frac{R_D}{\bar{V}} + T_D} \qquad\qquad \text{F-3}$$

The second tactic is to sprint to a new position such that the circles of detection by the sensor during drift do not overlap. A vehicle conducting a random search of this type (constrained by non-overlap) would spend a greater time sprinting in each cycle and would leave large random holidays in the area to be searched.

The objective is to compare the two tactics to determine the ratios of area searched in a given time.

To simplify the calculation, the non-overlap tactic used is a special case where the searcher sprints in a straight path a distance of $2R_D$, which results in consecutive detection circles which are tangent.

Thus, the time of a single search cycle is:

$$\frac{2R_D}{\bar{V}} + T_D$$

so that, in a total time T, there are:

$$\frac{T}{2\frac{R_D}{\bar{V}} + T_D} \quad \text{looks, and}$$

the total area searched is:

$$\frac{T \pi R_D^2}{2 \frac{R_D}{\tilde{V}} + T_D} \qquad \text{F-4}$$

Note that, in this special case, required sprinting time is the minimum for a "random" search subject to the constraint. Thus, ratios of the search rates of the two tactics are limiting calculations favoring the random case. (An indication of the sensitivity of search rate to this assumption appears in Table F-1.)

From equation F-4, the search rate for the random case is:

$$\frac{A'}{T} = \frac{\pi R_D^2}{2 \frac{R_D}{\tilde{V}} + T_D} \qquad \text{F-5}$$

Dividing equation F-3 by equation F-5, we obtain the ratio of the search rate of the tactic with overlap to that of the random tactic:

$$\frac{\sqrt{3}}{\pi} \left(1 + \frac{R_D}{R_D + T_D \tilde{V}} \right) \qquad \text{F-6}$$

Figure F-1 plots, for various values of sprint (flying) speed, combinations of drift time (T_D) and detection range (R_D) at which the ratio of search rates is unity. The accompanying discussion sheet provides detailed development and comparison. In general, for the assumed values of the parameters, the random tactic results in higher search rates.

Figure F-1

Combinations of Drift Time (T_D) and Detection Range (R_D)
Yielding Equal Search Rates for Overlap and Random Search Tactics
(Calculated for Various Sprint or Flying Speeds)

Detection Range While Drifting, R_D (nm)

Figure F-1

Combinations of Drift Time (T_D) and Detection Range (R_D)
Yielding Equal Search Rates for Overlap and Random Search Tactics
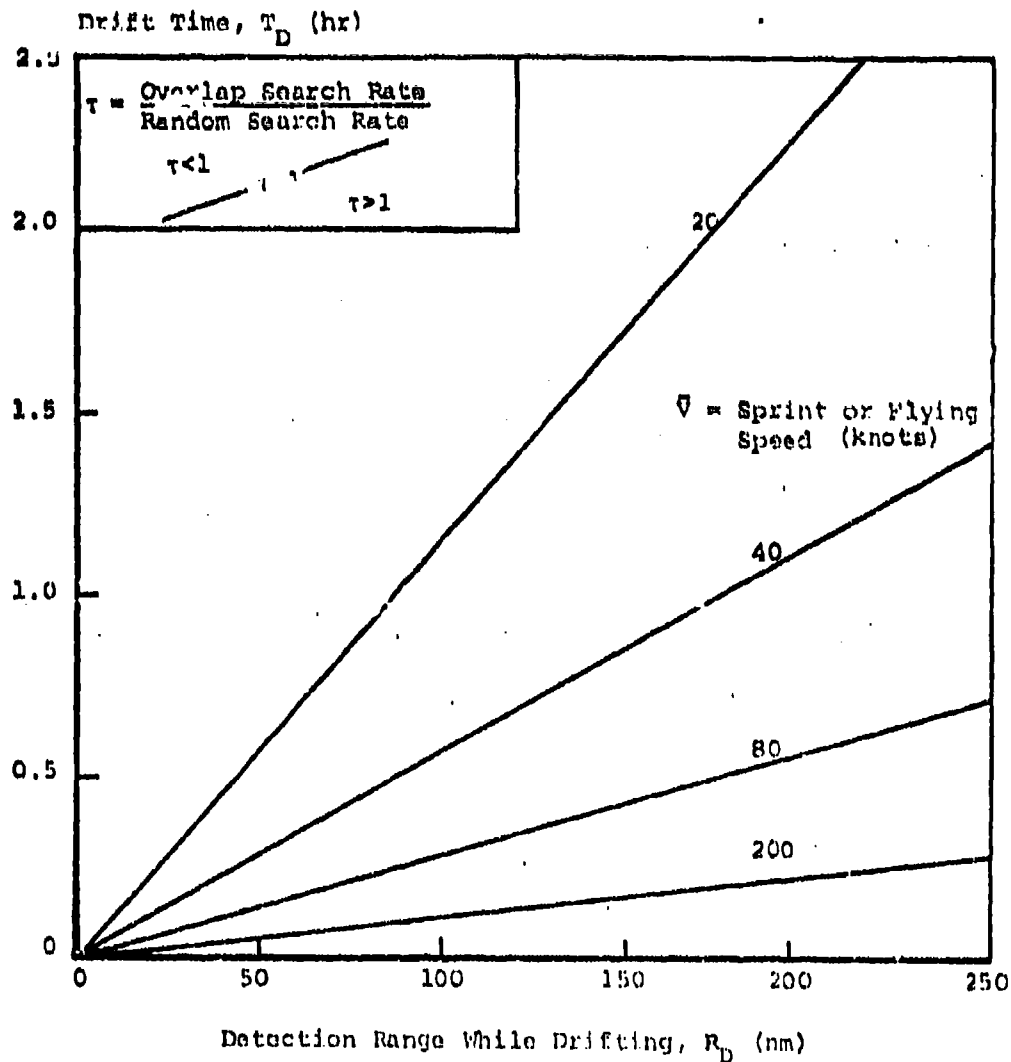**(Calculated for Various Sprint or Flying Speeds)**

Purpose

To indicate the relative search rates between:

- An overlapping search tactic along a path and

- A random search tactic

in sprint-drift or flying-drift search for various values of the pertinent

parameters.

Basis for Calculations

The figure is a plot of equal search rates for the two tactics for selected

sprint (or flying) speeds. That is, from Equation F-6:

$$\frac{\sqrt{3}}{\pi} \left(1 + \frac{R_D}{R_D + T_D \bar{V}} \right) = 1 \text{ (for each } \bar{V} \text{ indicated)}$$

where

$\bar{V}$ is the sprint (or flying) speed in knots

T_D is the drift time in hours required to complete a search period

(For sprint-drift, this includes time for the towed array to settle

and time to search. For flying-dirft, there is an additional drift

time required to stream the array before searching and to recover it

after the search.)

R_D is the detection range of the sensor (at zero speed) in nautical miles:

and the values of these parameters are the same for either tactic.

The figure plots combinations of R_D and T_D at which the value of the ratio of search rates is unity for the indicated values of $\bar{V}$. As indicated, for each $\bar{V}$, any combination of T_D and R_D above and to the left of the line is a case where the overlap search rate is less than that of the random search tactic.

Principal Points

1. Current technology indicates the following approximate combinations of the pertinent parameters:

	$\bar{V}$ (kts)	T_D (hr)	R_D (nm)
Sprint Drift	80	0.3	10-25
Flying Drift	200	1.5	10-25

As the figure indicates, the random search tactic produces higher search rates for these combinations.

2. There is, however, an artificiality in that the random tactic employed is a limiting case wherein the sprint (flying) distance ($2R_D$) is a minimum for a non-overlapping random search. The sensitivity of the results to this assumption was tested by considering a random search pattern wherein the average distance between search centers was doubled to $4R_D$ (still maintaining the constraint of non-overlap). Comparisons of the actual search rates are tabulated in Table F-1.

Table F-1

Type of Search	Actual Rates (nm^2/hr)	
	$R_D = 10$	$R_D = 25$

Sprint - Drift

$(\bar{V} = 80, \ T_D = 0.3)$

	$R_D = 10$	$R_D = 25$
Overlap	408	1767
Random		
Sprint Distance $= 2R_D$	570	2123
" " $= 4R_D$	392	1415

Flying-Drift

$(\bar{V} = 200, \ T_D = 1.5)$

	$R_D = 10$	$R_D = 25$
Overlap	112	666
Random		
Sprint Distance $= 2R_D$	196	1122
" " $= 4R_D$	185	1062

3.   In sprint-drift, the preferred tactic is sensitive to the average sprint distance required for the random search.   In flying-drift, random is clearly preferred because of the higher speed.

4.   The table indicates that for either tactic, sprint-drift produces much higher search rates than does flying-drift.   This is due to the much higher T_D/R_D ratio due to the assumption of $T_D = 1.5$ hours.   Since any gain in detection range (R_D) which may be possible should be equally available to sprint-drift vehicles, competitive flying-drift vehicles would require some combination of higher flying speeds, and shorter drift times than the value assumed ($T_D = 1.5$ hrs.).

5. Finally, it should be noted that the search rate criterion is not the final measure for comparison. If the specific scenario describing the search area and threat were developed, the respective search rates could be used to compute corresponding probabilities of detection.

G.   SEA LOITER (SEA SITTING) AIRCRAFT

This section of the appendix discusses sea loiter vehicles generally
and compares (for a general mission) force level requirements of such vehicles
with those of air loiter aircraft.

Currently available information* on sea loiter vehicle concepts indicates
the following:

Speed range	– 200-500 knots
Gross weight	– 500-1000 tons (C5A ≈ 350 tons)
Useful load (Payload plus fuel)	– 60%-70% (C5A ≈ 50%-60%)

The sea loiter concept implies a very large vehicle capable of high unit
payloads and long airborne endurance independent of the sea sitting characteris-
tic. Air loiter vehicle concepts have similar characteristics, except for the sea
sitting capability.

Data on such vehicles is sketchy and specific missions have not been
defined. In this analysis, we assume three general motivations for the
sea sitting capability:

1. Dramatic increase in total endurance resulting from the capability
    to sit in a condition involving very low fuel consumption for periods of
    up to several days. Under certain conditions, this may result in
    reduced force level requirements for a given posture (reduced
    Base Loss Factor).

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* ANVCE, Interim Evaluation Seminar, Air Vehicles Summary, Peter J. Mantle,
  Technical Director, 26 March 1976.

2. Short response times and capability for performing missions requiring
    high speeds.

3. Ability to utilize sensor systems not otherwise employable by aircraft.

Thus, the sea sitting concept makes it possible to combine the advantages
of the endurance (thus, lower BLF) and sensor capability of surface vehicles
with the rapid response (and surge) capability of aircraft.

However, unless the mission is such as to require both a surface vehicle
capability and rapid response, a continuing single station mission may be
equally fulfilled by a surface (or near surface) vehicle in one case or an
air loiter vehicle in the other.

Consider a mission which requires one vehicle continuously on a single
station. The mission is further specified in that, on activation by a detection
or on direction from base, there is a requirement for an aircraft to fly
continuously on station for an unspecified, but long, period of time.

In this case, throughout any active period, there is no sea sitting
and the ready force required to support the mission is identical to that of
a comparable air loiter vehicle force. There is no appreciable difference
in the BLF* during this period. The required force level of ready aircraft
would be the same. Thus, the only difference in total inventory requirements
would be that resulting from the reduced flying hours of ready sea loiter
aircraft during the non-active periods.

There are, however, potential missions where the force level requirements
for sea sitting aircraft could be much smaller. These occur when there is
a requirement for continuously occupying several such stations simultaneously

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*Defined as in Section A of the Appendix in terms of ready a/c only, that is
 a/c on station plus a/c in transit.

(such as in a long barrier line). If there is a high confidence that only a few of many stations might simultaneously be active, the sea sitting force level can be tailored accordingly. The air loiter force cannot.

This can be illustrated by a simple example. Parameters are as follows:

n = number of stations which must be simultaneously occupied

n_a = maximum expected number of simultaneously active stations

T_E = total endurance time of an aircraft (hr)

T_{Tr} = two way transit time (hr)

T_{St} = mission time, flying on station (hr)

T_S = sea sitting endurance time (fuel consumption assumed to be zero)

T_E = $T_{Tr} + T_{St}$ for air loiter aircraft

     = $T_{Tr} + T_{St} + T_S$ for sea loiter aircraft

Thus, as in Equation A-4, the required ready inventory of air loiter aircraft is:

$$n\left(\frac{T_{Tr} + T_{St}}{T_{St}}\right)$$

For the sea loiter aircraft, the requirement reduces to:

$$n_a\left(\frac{T_{Tr} + T_{St}}{T_{St}}\right) + (n-n_a)\left(\frac{T_{Tr} + T_S}{T_S}\right)$$

Whenever n is much larger than n_a and

        T_S is much larger than T_{St}

the force level requirement for sea sitters is much smaller.

Figure G-1 illustrates for a simple example where:

$$T_{Tr} = 10 \text{ hrs}$$

$$T_{St} = 10 \text{ hrs}$$

$$T_S = 100 \text{ hrs}$$

Figure G-1

Example of Potential Reductions in
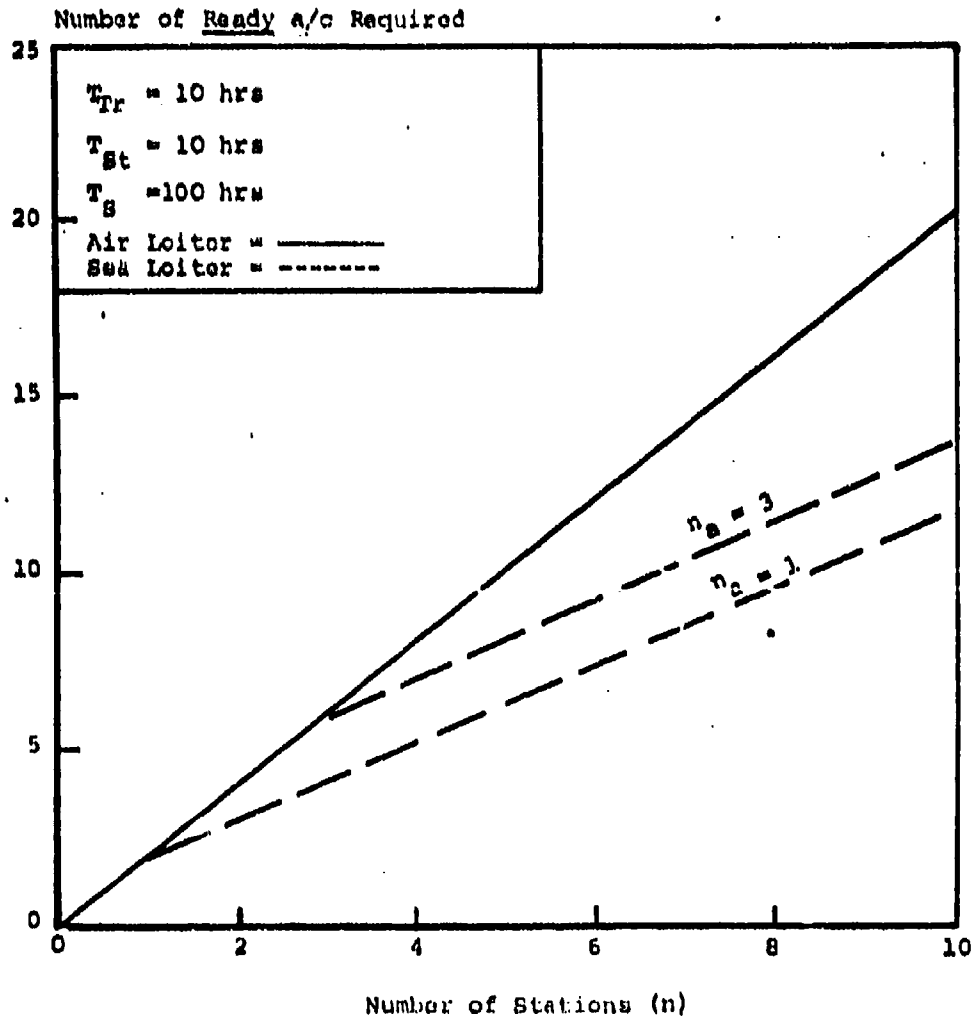Force Level Requirements Resulting From
Sea Loiter Capability

Number of Ready a/c Required

Number of Stations (n)

Figure G-1

Example of Potential Reductions in Force Level Requirements Resulting from Sea Loiter Capability

Purpose:

To illustrate the potential advantage of sea loiter aircraft with long sea sitting capability.

Basis of Calculations:

It is assumed that the maximum number of alert stations (n_a) is known with high confidence and that fuel consumption while sea sitting is essentially zero.

Let n = number of stations

$\quad n_a$ = maximum number of activated stations

$\quad T_E$ = total endurance time of an aircraft (hr)

so that

$$T_E = T_{Tr} + T_{St} \text{ for air loiter a/c, and}$$

$$T_E = T_{Tr} + T_{St} + T_S \quad \text{for sea loiter (where fuel consumption while sitting} = 0)$$

Thus, the number of ready aircraft required for air loiter is

$$n\left(\frac{T_{Tr} + T_{St}}{T_{St}} \right) \quad ,$$

and for sea loiter is

$$n_a\left(\frac{T_{Tr} + T_{St}}{T_{St}} \right) + (n - n_a)\left(\frac{T_{Tr} + T_S}{T_S} \right)$$

Principal Points:

1. As $(n - n_a)$ gets larger, the difference in force level requirements

gets larger. The BLF for air loiter remains constant; for sea loiter the BLF reduces as $(n - n_n)$ increases.

2. To the extent that sea loitering consumes fuel, these differences will be decreased. If the mission calls for continuous air operation commencing at activation, each activated sea sitter must take off with enough fuel to fly on station until his relief arrives, plus enough fuel to return to base - that is, enough fuel for a two-way transit if his relief must come from the base (less, if nearby stations can be temporarily vacated). In this case, sea sitting time must be reduced such that fuel remaining can always meet the on station flying requirement and the transit back to base.